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Effects Due to a Scalar Coupling on the Particle-Antiparticle Production in the Duffin-Kemmer-Petiau Theory

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Abstract The Duffin-Kemmer-Petiau formalism with vector and scalar potentials is used to point out a few misconceptions diffused in the literature. It is explicitly shown that the scalar coupling makes the DKP formalism not equivalent to the Klein-Gordon formalism or to the Proca formalism, and that the spin-1 sector of the DKP theory looks formally like the spin-0 sector. With proper boundary conditions, scattering of massive bosons in an arbitrary mixed vector-scalar square step potential is explored in a simple way and effects due to the scalar coupling on the particle-antiparticle production and localization of bosons are analyzed in some detail.

Keywords DKP equation · Klein's paradox · Pair production · Localization

1 Introduction

In a recent paper [8], scattering of massive spin-0 and spin-1 bosons under the influence of a vector smooth potential and a smooth position-dependent mass (that is to say, a scalar smooth potential) has been analyzed with the Duffin-Kemmer-Petiau (DKP) formalism. It has been shown that the boundary conditions imposed on the DKP spinor for a square step potential has to be obtained from those ones for a smooth step potential (see also [3]), that the DKP formalism is equivalent to the Klein-Gordon and to the Proca formalisms, and that the charge conservation law is violated under circumstances favorable to the existence of Klein's paradox.

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In the present paper, it is explicitly and precisely shown that the presence of a scalar coupling makes the DKP formalism not equivalent to the Klein-Gordon or to the Proca formalisms, that under the influence of scalar and vector one-dimensional potentials the spin-1 sector of the DKP theory looks formally like the spin-0 sector, that the proper boundary conditions imposed on the DKP spinor for a square step potential become evident without recurring to the limit of scattering in a smooth step potential and to Heun's function. Furthermore, effects due to a scalar coupling on the particle-antiparticle production are analyzed in some detail and it is pointed out that the charge is always conserved if one uses an acceptable definition of the reflection and transmission coefficients. An apparent paradox concerning the uncertainty principle is solved by introducing the concept of effective Compton wavelength. Comparison with the results obtained formerly with the Klein-Gordon formalism [1] highlights the differences between the DKP and the Klein-Gordon formalisms.

2 The DKP Equation

The first-order DKP equation for a massive free boson is given by [5]

$$(i\beta^{\mu}\partial_{\mu} - m)\psi = 0 \tag{1}$$

where the matrices β^{μ} satisfy the algebra

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\lambda\nu}\beta^{\mu} \tag{2}$$

and the metric tensor is $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. The algebra expressed by (2) generates a set of 126 independent matrices whose irreducible representations are a trivial representation, a five-dimensional representation and a ten-dimensional representation. The second-order Klein-Gordon and Proca equations are obtained when one selects the spin-0 and spin-1 sectors of the DKP theory. A well-known conserved four-current is given by

$$J^{\mu} = \bar{\psi}\beta^{\mu}\psi \tag{3}$$

where the adjoint spinor $\bar{\psi} = \psi^{\dagger}\eta^{0}$, with $\eta^{0} = 2\beta^{0}\beta^{0} - 1$ in such a way that $(\eta^{0}\beta^{\mu})^{\dagger} = \eta^{0}\beta^{\mu}$. Despite the similarity to the Dirac equation, the DKP equation involves singular matrices, the time component of J^{μ} given by (3) is not positive definite and the case of massless bosons cannot be obtained by a limiting process. Nevertheless, the matrices β^{μ} plus the unit operator generate a ring consistent with integer-spin algebra [6] and J^{0} may be interpreted as a charge density.

With the introduction of interactions, the DKP equation for a massive boson can be written as

$$(i\beta^{\mu}\partial_{\mu} - m - U)\psi = 0 \tag{4}$$

where the potential matrix U with scalar and vector terms is in the form

$$U = S + \beta^{\mu}A_{\mu} \tag{5}$$

with S and A_{μ} denoting the scalar and four-vector potential functions, respectively. Recently, by a proper interpretation of the DKP spinor components, it has been shown an anomalous term already noted by Kemmer [5] disappears from the DKP formalism so that the DKP equation and the Klein-Gordon and Proca equations are equivalent under minimal coupling [7, 11]. It is still true that J^μ is a conserved quantity in the presence of interactions expressed by (5) and that it can be interpreted as a charge current.

For the case of spin 0, we use the representation for the β^μ matrices given by [9]

$$\beta^0 = \begin{pmatrix} \theta & \overline{0} \\ \overline{0}^T & \mathbf{0} \end{pmatrix}, \qquad \beta^i = \begin{pmatrix} \widetilde{0} & \rho_i \\ -\rho_i^T & \mathbf{0} \end{pmatrix}, \qquad i = 1, 2, 3 \tag{6}$$

where

$$\begin{aligned} \theta &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \rho_1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \rho_2 &= \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \rho_3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{7}$$

$\overline{0}$, $\widetilde{0}$ and $\mathbf{0}$ are 2×3, 2×2 and 3×3 zero matrices, respectively, while the superscript T designates matrix transposition. The five-component spinor can be written as $\psi^T = (\psi_1, \dots, \psi_5)$ in such a way that the DKP equation for a boson constrained to move along the x-axis decomposes into

$$\begin{gathered} D_0\psi_1 = -i(m+S)\psi_2, \qquad D_1\psi_1 = -i(m+S)\psi_3, \\ D_0\psi_2 - D_1\psi_3 = -i(m+S)\psi_1, \\ \psi_4 = \psi_5 = 0 \end{gathered} \tag{8}$$

where

$$D_\mu = \partial_\mu + iA_\mu \tag{9}$$

and J^μ can be written as

$$J^0 = 2\,\mathrm{Re}(\psi_2^*\psi_1), \qquad J^1 = -2\,\mathrm{Re}(\psi_3^*\psi_1), \qquad J^2 = J^3 = 0. \tag{10}$$

It is worthwhile to note that $(D^\mu D_\mu + m^2)\psi_1 = 0$ in the absence of the scalar potential, so that the DKP equation reduces to the Klein-Gordon equation. The form $\partial_1 + iA_1$ in (8) suggests that the space component of the minimal vector potential can be gauged away by defining a new spinor

$$\tilde{\psi}(x,t) = \exp\left[i\int^x d\zeta\, A_1(\zeta,t)\right]\psi(x,t). \tag{11}$$

Then, without loss of generality, we will consider $A_1 = 0$.

For the case of spin 1, the β^μ matrices are [10]

$$\beta^0 = \begin{pmatrix} 0 & \overline{0} & \overline{0} & \overline{0} \\ \overline{0}^T & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \overline{0}^T & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \overline{0}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}, \qquad \beta^i = \begin{pmatrix} 0 & \overline{0} & e_i & \overline{0} \\ \overline{0}^T & \mathbf{0} & \mathbf{0} & -is_i \\ -e_i^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \overline{0}^T & -is_i & \mathbf{0} & \mathbf{0} \end{pmatrix} \tag{12}$$

where s_i are the 3×3 spin-1 matrices $(s_i)_{jk} = -i\varepsilon_{ijk}$, e_i are the 1×3 matrices $(e_i)_{1j} = \delta_{ij}$ and $\overline{0} = (0\;0\;0)$, while $\mathbf{I}$ and $\mathbf{0}$ designate the 3×3 unit and zero matrices, respectively. In the wake of previous works [3, 8]), the spinor $\psi^T = (\psi_1, \dots, \psi_{10})$ can be partitioned as

$$\psi_I^T = (\psi_3, \psi_4, \psi_5), \qquad \psi_{II}^T = (\psi_6, \psi_7, \psi_2), \qquad \psi_{III}^T = (\psi_{10}, -\psi_9, \psi_1) \tag{13}$$

so that the one-dimensional DKP equation can be expressed in the form

$$D_0\psi_I = -im\psi_{II}, \qquad D_1\psi_I = -im\psi_{III},$$
$$D_0\psi_{II} - D_1\psi_{III} = -i(m+S)\psi_I, \tag{14}$$
$$\psi_8 = 0$$

where D_μ is again given by (9). In addition, expressed in terms of (13) the current can be written as

$$J^0 = 2\,\mathrm{Re}(\psi_{II}^\dagger\psi_I), \qquad J^1 = -2\,\mathrm{Re}(\psi_{III}^\dagger\psi_I), \qquad J^2 = J^3 = 0. \tag{15}$$

Comparison of (8) with (14) evidences that the spinors ψ_I, ψ_{II} and ψ_{III} behave like the spinor components ψ_1, ψ_2 and ψ_3, respectively, from the spin-0 sector of the DKP theory. More than this, comparison of (10) with (15) places on view that the spin-1 sector of the DKP theory looks formally like the spin-0 sector.

According to the observation of the last paragraph, we will restrict our attention to the spin-0 sector of the DKP theory. If the terms in the potential U are time-independent, one can write $\psi(x,t) = \varphi(x)\exp(-iEt)$ in such a way that the time-independent DKP equation for the spin-0 sector splits into

$$(m+S)\frac{d}{dx}\left(\frac{1}{m+S}\varphi_1'\right) + K^2\varphi_1 = 0,$$
$$\varphi_2 = \frac{E-A_0}{m+S}\varphi_1, \tag{16}$$
$$\varphi_3 = \frac{i}{m+S}\varphi_1'$$

where the prime denotes derivative with respect to x and

$$K^2 = (E-A_0)^2 - (m+S)^2. \tag{17}$$

For this time-independent problem, J^μ has the components

$$J^0 = 2\frac{E-A_0}{m+S}|\varphi_1|^2, \qquad J^1 = 2\frac{\mathrm{Im}(\varphi_1'\varphi_1^*)}{m+S}. \tag{18}$$

Since J^μ is not time dependent, φ describes a stationary state.

Just as in the case of a pure vector coupling [2], there is no reason to require that the spinor and its derivative are continuous across finite discontinuities of the potential, as naively advocated in [3, 8]. A careful analysis reveals, though, that proper matching conditions follow from the differential equations obeyed by the spinor components, as they should be, avoiding in this manner to recur to the limit process of smooth potentials. The effect of the discontinuity of the potential can be evaluated by integrating the equations for the components of the DKP spinor from $-\delta$ to $+\delta$, by supposing that $x=0$ is the point of interest, and taking the limit $\delta \to 0$. In fact, the second-order differential equation given by the third line of (16) implies that φ_1 is continuous and the first line implies that so is $\varphi_1'/(m+S) = -i\varphi_3$. In this case, φ_2 is discontinuous and so is J^0, but not J^1. A possible discontinuity of J^0 would not matter if it is to be interpreted as a charge density but J^1 (involving $\varphi_1^*\varphi_1'/(m+S)$) should be continuous in a stationary regime.

3 The Step Potential

The one-dimensional square step potential is expressed as

$$S=\theta(x)c_S V, \qquad A_0=\theta(x)c_A V \tag{19}$$

where c_S and c_A are dimensionless and positive coupling constants constrained by $c_S+c_A=1$, $\theta(x)$ denotes the Heaviside step function and $V>0$ is the height of the step. For $x<0$ the DKP equation has the solution

$$\varphi(x)=\varphi_+e^{+ikx}+\varphi_-e^{-ikx} \tag{20}$$

where

$$\varphi_\pm^T=\frac{a_\pm}{\sqrt{2}}\left(1,\frac{E}{m},\mp\frac{k}{m},0,0\right) \tag{21}$$

and

$$k=\sqrt{E^2-m^2}. \tag{22}$$

For $|E|>m$, the solution expressed by (20) and (21) describes plane waves propagating on both directions of the x-axis. The flux related to the current J^μ, corresponding to φ given by (20), is expressed as

$$J^1=\frac{k}{m}\left(|a_+|^2-|a_-|^2\right) \tag{23}$$

and

$$J_\pm^0=\frac{E}{m}|a_\pm|^2. \tag{24}$$

If we choose incident particles on the potential barrier ($J^0>0$), $\varphi_+\exp(+ikx)$ will describe incident particles ($J^1>0$), whereas $\varphi_-\exp(-ikx)$ will describe reflected particles ($J^1<0$). On the other hand, for $x>0$ the solution describes an evanescent wave or a progressive wave running away from the potential interface. The general solution has the form

$$\varphi_{\mathrm{t}}(x)=(\varphi_{\mathrm{t}})_+e^{+iqx}+(\varphi_{\mathrm{t}})_-e^{-iqx} \tag{25}$$

where

$$(\varphi_{\mathrm{t}})_\pm^T=\frac{b_\pm}{\sqrt{2}}\left(1,\frac{E-c_AV}{m+c_SV},\frac{\mp q}{m+c_SV},0,0\right) \tag{26}$$

and

$$q=\sqrt{(E-c_AV)^2-(m+c_SV)^2}. \tag{27}$$

Due to the twofold possibility of signs for the energy of a stationary state, the solution involving b_- cannot be ruled out a priori. As a matter of fact, this term may describe a progressive wave with a negative charge density and a negative flux of charge ($J^1<0$). In other words, the solution $(\varphi_{\mathrm{t}})_-\exp(-iqx)$ with $q\in\mathbb{R}$ reveals a signature of Klein's paradox. One can readily envisage that three different classes of solutions can be segregated:

- Class A. For $V < E - m$ one has $q \in \mathbb{R}$, and the solution describing a plane wave propagating in the positive direction of the x-axis is possible only if $b_- = 0$. In this case the components of the current are given by

$$J^0 = \frac{E - c_A V}{m + c_S V}|b_+|^2, \qquad J^1 = \frac{q}{m + c_S V}|b_+|^2. \tag{28}$$

- Class B. For $E - m < V < V_c$, where

$$V_c = \begin{cases} \frac{E+m}{2c_A - 1}, & \text{for } c_A > 1/2, \\ \infty, & \text{for } c_A \leq 1/2 \end{cases} \tag{29}$$

one has that $q = +i|q|$ or $q = -i|q|$. The solution with $q = \pm i|q|$ demands $b_\mp = 0$ for furnishing a finite charge density as $x \to \infty$. In this case $J^1 = 0$ and

$$J^0 = \begin{cases} \frac{E - c_A V}{m + c_S V} e^{-2|q|x}|b_+|^2, & \text{for } q = +i|q| \ (V < E/c_A), \\ -\frac{c_A V - E}{m + c_S V} e^{-2|q|x}|b_-|^2, & \text{for } q = -i|q| \ (V > E/c_A). \end{cases}$$

- Class C. With $V > V_c$ it appears again the possibility of propagation in the positive direction of the x-axis, now with $b_+ = 0$. The current takes the form

$$J^0 = -\frac{c_A V - E}{m + c_S V}|b_-|^2, \qquad J^1 = -\frac{q}{m + c_S V}|b_-|^2. \tag{30}$$

The demand for continuity of φ_1 and $\varphi_1'/(m+S)$ at $x = 0$ fixes the wave amplitudes in terms of the amplitude of the incident wave, viz.

$$\frac{a_-}{a_+} = \begin{cases} \frac{k - \tilde{q}}{k + \tilde{q}} & \text{for the class A}, \\ \frac{(k - i|\tilde{q}|)^2}{k^2 + |\tilde{q}|^2} & \text{for the class B}, \\ \frac{k + \tilde{q}}{k - \tilde{q}} & \text{for the class C}, \end{cases} \tag{31}$$

$$\frac{b_+}{a_+} = \begin{cases} \frac{2k}{k + \tilde{q}} & \text{for the class A}, \\ \frac{2k(k - i|\tilde{q}|)}{k^2 + |\tilde{q}|^2} & \text{for the class B}, \\ 0 & \text{for the class C}, \end{cases} \tag{32}$$

$$\frac{b_-}{a_+} = \begin{cases} 0 & \text{for the class A}, \\ 0 & \text{for the class B}, \\ \frac{2k}{k - \tilde{q}} & \text{for the class C} \end{cases} \tag{33}$$

where $\tilde{q} = q(1 + c_S V/m)^{-1}$. Now we focus attention on the calculation of the reflection (R) and transmission (T) coefficients. The reflection (transmission) coefficient is defined as the ratio of the reflected (transmitted) flux to the incident flux. Since $\partial J^0/\partial t = 0$ for stationary states, one has that J^1 is independent of x. This fact implies that

$$R = \begin{cases} \left(\frac{k - \tilde{q}}{k + \tilde{q}}\right)^2 & \text{for the class A}, \\ 1 & \text{for the class B}, \\ \left(\frac{k + \tilde{q}}{k - \tilde{q}}\right)^2 & \text{for the class C}, \end{cases} \tag{34}$$

$$T = \begin{cases} \frac{4k\tilde{q}}{(k+\tilde{q})^2} & \text{for the class A,} \\ 0 & \text{for the class B,} \\ -\frac{4k\tilde{q}}{(k-\tilde{q})^2} & \text{for the class C.} \end{cases} \tag{35}$$

It is instructive to note that (31)–(35) look like those ones for the mixed vector-scalar square step potential in the Klein-Gordon formalism [1]. Interestingly, $\tilde{q} = q(1 + c_S V/m)^{-1}$ in the DKP formalism whereas $\tilde{q} = q$ in the Klein-Gordon formalism. The expression for $\tilde{q}$ departs from q just by the factor $(1 + c_S V/m)^{-1}$. It is clear that the scalar coupling makes all the difference, even in the nonrelativistic limit.

In the class C we meet a bizarre circumstance as long as both J^0 and J^1 are negative quantities. It is satisfactory to interpret the solution $(\varphi_t)_- \exp(-iqx)$ as describing the propagation, in the positive direction of the x-axis, of particles with charges of opposite sign to the incident particles. This interpretation is consistent if the particles moving in this region have energy $-E$ and are under the influence of a potential $-c_A V$. It means that, in fact, the progressive wave describes the propagation of antiparticles in the positive direction of the x-axis. For all the classes one has $R + T = 1$ as should be expected for a conserved quantity. The class C presents $R > 1$, the alluded Klein's paradox, implying that more particles are reflected from the potential barrier than those incoming. It must be so because, as seen before, the potential stimulates the production of antiparticles at $x = 0$. Due to the charge conservation there is, in fact, the creation of particle-antiparticle pairs. Since the potential in $x > 0$ is repulsive for particles they are necessarily reflected. From the previous discussion related to the classes B and C, one can realize that the threshold energy for the pair production is given by $V = V_c$ for $c_A > 1/2$ and that for $c_A \leq 1/2$ the pair production is not feasible. Evidently, the scalar coupling increases the minimal energy necessary for the pair production. The minimum value for the threshold ($V = 2m$) occurs when there is a pure vector coupling ($c_A = 1$). The addition of a scalar contaminant contributes for increasing the threshold, which surprisingly becomes infinity for a half-and-half admixture of couplings. Then, the pair production is not workable if the vector coupling does not exceed the scalar one, even if the V is extremely strong. The propagation of antiparticles inside the potential barrier can be interpreted as due to the fact that each antiparticle is under the influence of an effective potential given by $(c_S - c_A)V$. In this way, each antiparticle has an available energy (rest energy plus kinetic energy) given by $(2c_A - 1)V - E$, accordingly one concludes about the threshold energy. One can also say that the particles are under the influence of an ascending step of height $(c_S + c_A)V$, and that the antiparticles are under the influence of an effective step of height $(c_S - c_A)V$, an ascending step (repulsive) if $c_A < 1/2$ and an descending step (attractive) if $c_A > 1/2$. For $E - m < V < V_c$ (class B), one has that $J^0 \gtrless 0$ for $V \lessgtr E/c_A$, thus the evanescent wave with $q = +i|q|$ ($q = -i|q|$) is related to particles (antiparticles). One can say that there is a charge polarization due to the vector potential. The maximum charge density for antiparticles occurs for $V = V_c$ and beyond this value they are pulled apart. For the class B, the charge density beyond the potential barrier is proportional to $\exp(-2|q|x)$ so that the uncertainty in the position in the region $x > 0$, estimated as being the value of x that makes the charge density equal to $J^0(0)/e$, is given by $\Delta x = 1/(2|q|)$. This uncertainty presents the minimum value

$$(\Delta x)_{\min} = \frac{1}{2(m + c_S V)} \tag{36}$$

when V becomes $V = E/c_A$. From this last result one can see that $(\Delta x)_{\min} = \lambda/2$ ($\lambda = 1/m$ is the Compton wavelength) in the case of a pure vector potential ($c_S = 0$). However, one can conclude that $(\Delta x)_{\min} < \lambda/2$ in the case of a vector potential contaminated with some scalar coupling. Furthermore, the penetration of the boson into the region $x > 0$ shrinks without limit with increasing V. At first glance it seems that the uncertainty principle dies away provided such a principle implies that it is impossible to localize a particle into a region of space less than half of its Compton wavelength (see, e.g., [4, 12]). This apparent contradiction can be remedied by recurring to the concepts of effective mass and effective Compton wavelength. Indeed, (36) suggests that we can define the effective mass as $m_{\mathtt{eff}} = m + c_S V$ in such a way that $(\Delta x)_{\min} = \lambda_{\mathtt{eff}}/2$ and $(\Delta p)_{\max} = m_{\mathtt{eff}}$, where the effective Compton wavelength is defined as $\lambda_{\mathtt{eff}} = 1/m_{\mathtt{eff}}$. It means that the localization of the boson does not require any minimum value in order to ensure the single-particle interpretation of the DKP equation.

4 Conclusions

We have explored the influence of scalar and vector interactions in the DKP formalism. We have shown that the spin-1 sector of the DKP theory looks formally like the spin-0 sector and that the scalar coupling makes the DKP formalism not equivalent to the Klein-Gordon or to the Proca formalisms. With proper boundary conditions imposed on the DKP spinor for a square step potential, we have analyzed in a very simple way the effects due to a scalar coupling on the particle-antiparticle production and on the localization of bosons. Another important conclusion of our work is that with an acceptable definition of the reflection and transmission coefficients the charge is not violated, even if Klein's paradox shows its face, and that the localization of the boson does not require any minimum value in the context of the single-particle interpretation of the DKP equation.

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